MATH-2400 Fall '05 Exam #2

1(a) (9p.). Consider the differential equation $y''-4y'+5y = e^{2t}\sin t+t^2$. Find the general *real* solution of the homogenous equation. Determine a suitable form for a particular solution $Y(t)$ if the method of undetermined coefficients is to be used. Do **NOT** evaluate the coefficients. **Ans:** $Y = A_1te^{2t}\sin t + B_1te^{2t}\cot s + C_0t^2 + C_1t + C_2$

(b). (4 pts) **Find** a particular solution of $y'' + 4y = 2e^{2t} + 1$. **Ans:** $Y = (1/4)(e^{2t} + 1)$

2(a) (6p.) Find eigenvalues and eigenvectors of the matrix $\begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$ **Ans**: $\lambda = -2 \pm i; \xi = \begin{bmatrix} 1 \mp i \\ 1 \end{bmatrix}$

(b) (8pts) Use the method of variation of parameters to find a particular solution of the equation $y'' + 2y' + y = -2e^{-t}/t$.

Ans: $Y = -2te^{-t}\ln t + [c_1e^{-t} + c_2te^{-t}]$ Addition of any homogemeous solution still leaves it a particular solution].

3.(a) (9 pts) The matrix $A = \begin{bmatrix} 1 & -1 \\ 5 & 3 \end{bmatrix}$ has $2+2i$ as eigenvalue with the corresponding eigenvector $\begin{bmatrix} 1 \\ -1-2i \end{bmatrix}$. For the system $\mathbf{x}' = A\mathbf{x}$

(i) Write down a *real* general solution. **Ans**:

$$C_1e^{2t}\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\cos 2t - \begin{bmatrix} 0 \\ -2 \end{bmatrix}\sin 2t\right) + C_2e^{2t}\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\sin 2t + \begin{bmatrix} 0 \\ -2 \end{bmatrix}\cos 2t\right)$$

(ii) Solve the initial value problem with $x_\mathbf{1} = 1, x_\mathbf{2} = 1$. **Ans:** $C_1 = 1, C_2 = -1$.

(iii) In the $x_\mathbf{1}x_\mathbf{2}$-plane (phase plane) sketch the solution curve (trajectory) passing through the point $(1, 1)$. **Ans:** Solution is spiraling to ∞ counter clockwise.

3.(b) (4 pts) Rewrite the 2nd order differential equation $y'' + t\sin y = t^2$ as a system of two first order equations. **Ans:** $x_\mathbf{1}' = x_2, x_\mathbf{2}' = -t\sin x_1 - t^2$

4. (10p.) The matrix $\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ has eigenvalues -3 and -1 with the corresponding eigenvectors $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. (i) Solve the initial-value problem:

$$\frac{dx}{dt} = -2x + y, \quad \frac{dy}{dt} = x - 2y; \quad x(0) = 3,\ y(0) = -2,$$

$$\mathbf{Ans}: \begin{bmatrix} x \\ y \end{bmatrix} = \frac{5}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}e^{-3t} + \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{-t}$$

(ii) In the xy-plane sketch the phase portrait of the system. **Ans:** It is a sink node.